DSP

19/10/2015 الاثنين

م. شهاب [3] سكشن

* Convolution Properties :-

① Commutation

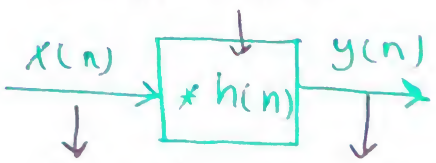
② Causality

$h(n)=0 \, , \, n<0$

③ Stability

$$\sum_{-\infty}^{\infty} |h(n)| < \infty$$

bounded لازم يكون



Stable ; Bounded I/P Bounded O/P

* for stability h(n) is bounded ✗ ← غير كافي

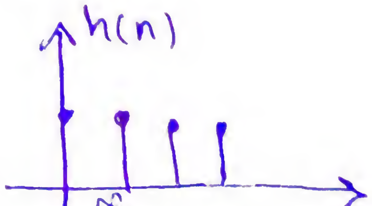
$$y(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$$

✓✓ # for stability

$$\sum_{n=-\infty}^{\infty} h(n) < \infty$$

ex:

$$\sum_{-\infty}^{\infty} h(n) = \infty$$



$$\sum_{-\infty}^{\infty} h(n) = 4$$

ex: $h(n) = u(n) - u(n-a)$

$$\sum_{-\infty}^{\infty} h(n) = a$$

$\Rightarrow$ stable

ex: $h(n) = u(n)$

$$\sum_{-\infty}^{\infty} h(n) = \infty$$

$\Rightarrow$ unstable

* Z - transform

على الهامش

Continous ⟶ Biscrete

differential Equations $\left(\frac{d}{dt}\right)$ → for Higher orders → Laplace Transform

difference Equation (Δ) → Z-transform

$$L[f(t)] = F(s) = \int_0^{\infty} f(t)\, e^{-st}\, dt$$

$$F(s) = \sum_0^{\infty} f(nT)\, e^{-snT}$$

$$z = e^{sT}$$

$$F(z) = \sum_0^{\infty} f(n)\, z^{-n}$$

Sheet:

① $x(n) = n e^{3n} u(n)$

find $X(z)$

$$X(z) = \sum_{n=0}^{\infty} x(n) z^{-n}$$

$$= \sum_{n=0}^{\infty} n e^{3n} z^{-n}$$

$$= e^{3}z^{-1} + 2e^{6}z^{-2} + 3e^{9}e^{-3} + 4e^{12}z^{-4} + \cdots$$

$$= e^{3}z^{-1}(1 + 2e^{3}z^{-1} + 3e^{6}z^{-2} + 4e^{9}z^{-3} + -)$$

$$= e^{3}z^{-1}(1 + 2e^{3}z^{-1} + 3(e^{3}z^{-1})^{2} + 4(e^{3}z^{-1})^{3} + \cdots$$

$$X(z) = \frac{e^{3}z^{-1}}{(1 - e^{3}z^{-1})^{2}}$$

Another Solution

$$X(z) = e^{3}z^{-1} + 2e^{6}z^{-2} + 3e^{9}z^{-3} + \cdots$$

$$e^{3}z^{-1}X(z) = e^{6}z^{-1} + 2e^{9}z^{-2} + 3e^{12}z^{-3} + \cdots$$

$$X(z) - e^{3}z^{-1}X(z) = e^{3}z^{-1} + e^{6}z^{-2} + e^{9}z^{-3} + \cdots$$

$$= e^{3}z^{-1}(1 + e^{3}z^{-1} + e^{6}z^{-2} + \cdots)$$

$$(1 - e^{3}z^{-1})X(z) = e^{3}z^{-1}\frac{1}{1 - e^{3}z^{-1}}$$

$$X(z) = \frac{e^{3}z^{-1}}{(1 - e^{3}z^{-1})^{2}}$$

② $x(n) = (-1)^{n} u(n)$

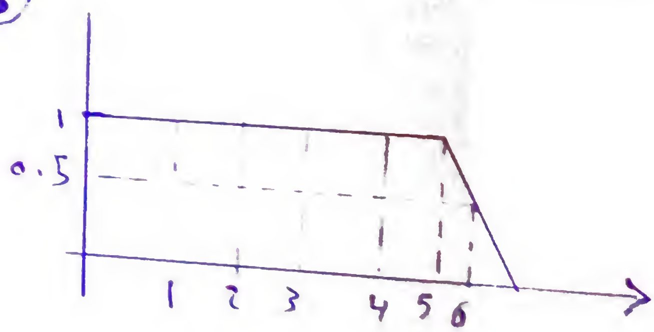
$$X(z) = \sum_{0}^{\infty} (-1)^{n} z^{-n}$$

$$= 1 + (-1)z^{-1} + (-1)^{2}z^{-2} + (-1)^{3}z^{-3} + \cdots$$

$$= 1 - z^{-1} + z^{-2} - z^{-3} + z^{-4} \cdots$$

$$= \frac{1}{1 + z^{-1}}$$

③

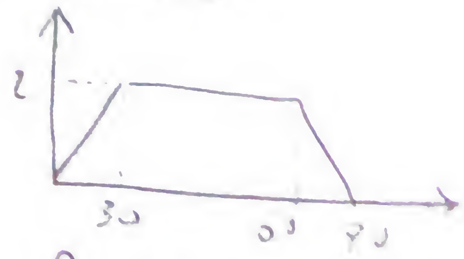


$x(n) = \{1, 1, 1, 1, 1, 0.5, 0\}$

$$X(z) = \sum_{0}^{\infty} x(n) z^{-n}$$

$$= 1 + z^{-1} + z^{-2} + z^{-3} + z^{-4} + 0.5 z^{-5}$$

Report



find $X(z)$ $T = 1$

Properties of Z-transform

① $Z[a\,x(n)] = a\,X(z)$

② $Z[x_1(n + x_2(n)] = X_1(z) + X_2(z)$

③ $Z[e^{\pm an} x(n)] = X(z)\Big|_{z = z e^{\mp an}}$ عكس الإشارة

④ $Z[n\,x(n)] = -z \frac{d}{dz} X(z)$

⑤ $Z[a^n\,x(n)] = X(z)\Big|_{z = z/a}$

⑥ $Z[x_1(n) * x_2(n)] = X_1(z) \cdot X_2(z)$

↑ Convolution

⑦ $Z[x(n \pm m)] = z^{\pm m} X(z)$

(الإشارة لا تتغير)

$x(n) = (0.5)^n + (0.25)^n + 3\delta(n-2)$

$* u(n)$

Find $X(z)$

$$X(z) = \frac{z}{z-1}\Big|_{z = \frac{z}{0.5}} + \frac{z}{z-1}\Big|_{z = z/0.25} + 3z^{-2}$$

$$= \frac{z}{z - 0.5} + \frac{z}{z - 0.25} + 3z^{-2}$$

$x(n) = (-1)^n \sin(\omega n) + e^{-2n} \cos(\omega n)$

$$X(z) = Z[\sin(\omega n)]\Big|_{z=-z} + Z[\cos(\omega n)]\Big|_{z = z e^{2}}$$

(عوض)

$x(n) = (-1)^n u(n)$

$$X(z) = U(z)\Big|_{z=-z}$$

$$= \frac{z}{z-1}\Big|_{z=-z}$$

$$= \frac{-z}{-z-1}$$

$$= \frac{z}{z+1} = \frac{1}{1+z^{-1}}$$

$$X(s) = \frac{10}{(s+1)(s+2)}$$

$$X(s) = \frac{10}{s+1} - \frac{10}{s+2}$$

$L^{-1}[X(s)] = x(t) = 10e^{-t} - 10e^{-2t}$

⇒ sampling $t = nT$, $T = 1$

$x(n) = 10e^{-n} - 10e^{-2n}$

$$X(z) = 10\left[\frac{z}{z-1}\Big|_{z e^{n}} - \frac{z}{z-1}\Big|_{z e^{2n}}\right]$$

حل آزمن

$$z = e^{sT}$$

$$\ln z = s\not{T}_1$$

$$X(z) = \frac{10}{(\ln z + 1)(\ln z + 2)}$$

$$y(n) = x_1(n) * x_2(n)$$

$$x_1(n) = (0.5)^n \, u(n-1)$$

$$x_2(n) = \delta(n) \, e^{2n}$$

Find $Y(z)$

$$Y(z) = X_1(z) \cdot X_2(z)$$

$$X_1(z) = \mathcal{Z}\left[U(z-1)\right]\Big|_{z = \frac{z}{0.5}}$$

$$= z^{-1} \mathcal{Z}\left[U(n)\right]\Big|_{z = \frac{z}{0.5}}$$

$$= \frac{\cancel{z^{-1}}\, \cancel{z}}{z-1}\Big|_{z = \frac{z}{0.5} = 2z}$$